THERMAL LANDAU JET

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The author considers the steady-state temperature distribution in a jet of incompressible fluid emitted in pulsed bursts from a point source, with and without account taken of viscous dissipation. When viscous dissipation is not taken into account, an exact solution is obtained for the energy equation, bounded in the entire closed interval $0 \le \theta \le \pi$. When viscous dissipation is considered, the energy equation is obtained using the introduced separation of variables.

§1. Consider the temperature distribution in an axisymmetric jet of incompressible fluid leaving a thin tube and entering an unbounded space filled with the same fluid. A point heat source of intensity $2\pi q_0$ is placed in the tube. We assume the source intensity is such that the change in density as a function of temperature can be ignored.

We write the energy equation of the jet without allowing for viscous dissipation for the steady-state temperature distribution:

$$\operatorname{div}(\mathbf{v}T - a\nabla T) = Q_0 \frac{1}{\rho c_p}, \tag{1}$$

where $Q_0 = 2\pi q_0 \delta(r)$ and $\delta(r)$ is the delta function.

Heat flow at the coordinate origin is not considered because of the infinitely large quantity of heat liberated. From (1) and from the Ostrogradskii-Gauss theorem, we obtain

$$\iint\limits_S \left(v_r T - a\frac{\partial T}{\partial r}\right) dS = 2\pi q_0 \frac{1}{\rho c_p}. \tag{2}$$

Using Eq. (2) and allowing for the values of the jet velocity components obtained in [1], we write the jet temperature as

$$T = \frac{y(\theta)}{r}. \tag{3}$$

(The temperature at infinity is assumed to be zero.) Condition (2) and the condition that T be restricted over the entire closed interval $0 \le \theta \le \pi$ are used as the boundary conditions for Eq. (1).

By writing Eq. (1) in spherical coordinates and introducing the new variable $x = \cos\theta$, we finally obtain

$$(1-x^2)y'' - 2xy' = 2\lambda(fy)', \tag{4}$$

allowing for (3) and the values for the velocity components v_r and v_θ (the primes denote differentiation with respect to x); here $\lambda \equiv \nu/a$ (this quantity is the Prandtl number);

$$f = \frac{1-x^2}{A-x} \tag{5}$$

($A \ge 1$ is a constant associated with the total momentun flux of the jet).

On integrating this equation and allowing for the boundary conditions, we obtain

$$y = \frac{C_1}{(A-x)^{2\lambda}}.$$

Defining the constant C_1 from condition (2), we obtain a final expression for the temperature

$$T = - q_0\lambda(2\lambda+1)\left(\frac{A^2-1}{A-\cos\theta}\right)^{2\lambda} \times$$
$$\times \left\{\nu r\rho c_p\left[(A+1)^{2\lambda}(A-4\lambda-1) - \right.\right.$$
$$\left.\left. - (A-1)^{2\lambda}(A+4\lambda+1)\right]\right\}^{-1}. \tag{6}$$

Consider the following three cases:

a) $2\lambda \ll A$. It follows from expression (6) that the temperature is

$$T = \frac{q_0\lambda}{2\nu r\rho c_p}\left(1 + \frac{2\cos\theta}{A}\right). \tag{7}$$

For a fluid with $\lambda \sim 1$, Eq. (7) becomes

$$T_{\lambda\approx 1} = \frac{q_0}{\nu r\rho c_p}\left(\frac{A+2\cos\theta}{2A}\right).$$

If $(2\cos\theta)/A \ll 1$ Eq. (6) becomes

$$T = \frac{q_0\lambda}{2\nu r\rho c_p}, \tag{8}$$

i.e., for a fluid with $2\lambda \ll A$ and low momentum ($A \gg 1$), the jet temperature does not depend on momentum or the polar angle θ but is a function only of the viscosity and radius.

b) $A \to 1$ (the momentum $P \to \infty$).

It follows from (6) that $T \to 0$. This means that the temperature approaches the temperature at infinity everywhere except at the flow axis $\theta = 0$, where the temperature is given by

$$T_0 = \frac{q_0(2\lambda+1)}{4\nu r\rho c_p}, \tag{9}$$

i.e., in this case the temperature at the axis undergoes a discontinuity, remaining bounded. From (7) and (9) it is clear that the temperature at the flow axis increases with increase in jet momentum.

c) $A \to 1$ (momentum approaches infinity); $\lambda \to 0$ ($\nu \to 0$; a is bounded). It then follows from (6) that

$$T = \frac{q_0}{2ra\rho c_p}. \tag{10}$$

For a fluid in which the viscosity approaches zero and the thermal diffusivity is finite for an infinite jet momentum, the jet temperature will not depend on the momentum or on the polar angle.

§2. If we allow for dissipation losses, the energy equation for the steady-state temperature distribution in a submerged jet now becomes

$$\mathrm{v}\nabla T - a\Delta T = \Phi + 2\pi\delta(r)\frac{q_0}{\rho c_p}, \tag{11}$$

where

$$\Phi = \frac{\nu}{2c_p}\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right)\frac{\partial v_i}{\partial x_k}.$$

The obtained equation (11) is linear. Its general solution is equal to the sum of the general solution of the homogeneous equation (5) obtained in §1 and the particular solution of Eq. (11).

Using (11) we write the particular solution as

$$T = \frac{z(\theta)}{r^2}. \tag{12}$$

We write Eq. (11) in spherical coordinates. Using the expressions for v_r and v_θ from [1], together with Eq. (4), we obtain

$$(1-x^2)z'' - 2xz' - 2\lambda fz' - 4\lambda f'z + 2z = -\Phi, \tag{13}$$

where

$$\Phi = \frac{8\nu}{\rho c_p} \times$$

$$\times\left[\frac{(1-2Ax+x^2)^2[(1-x^2)+(1+A^2-2Ax)]}{(A-x)^6} - \frac{Ax-1}{(A-x)^4} + \frac{x^3}{(A-x)^2}\right].$$

It is very difficult to evaluate the particular solution of Eq. (13). We can show that for a homogeneous equation corresponding to (13), $\lambda = 0$ and $\lambda = 1/2$ (for A = 1) are eigenvalues.

However, these two cases have no physical meaning, since $\lambda = 0$ means that the viscosity is zero, while, when A = 1, the velocity components v_r and v_θ undergo a discontinuity at the $\theta = 0$ axis.

NOTATION

T is the temperature; ν is the kinematic viscosity of the fluid; ρ is the fluid density; $\lambda = \nu_r$ is the Prandtl number; $a = k/\rho c_p$ is the thermal diffusivity; c_p is the specific heat of the fluid; v_r is the radial velocity component; v_θ is the polar velocity component; Φ is a dissipative function; k is the specific thermal conductivity.

REFERENCES

1. L. D. Landau and E. M. Lifshits, Mechanics of Continuous Media [in Russian], §23, GITTL, 1954.

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